THE INTERSECTION OF THREE SPHERES IN A SPHERE AND A NEW APPLICATION OF THE SATO-LEVINE INVARIANT

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Abstract. Take transverse immersions $f: S^4_1 \amalg S^4_2 \amalg S^4_3 \looparrowright S^6$ such that (1) $f|S^4_i$ is an embedding, (2) $f(S^4_i) \cap f(S^4_j) \neq \phi$ and $f(S^4_i) \cap f(S^4_j)$ is connected, and (3) $f(S^4_1) \cap f(S^4_2) \cap f(S^4_3) = \phi$. Then we obtain three surface-links L_i= $(f^{-1}(f(S^4_i) \cap f(S^4_j)),\ f^{-1}(f(S^4_i) \cap f(S^4_k))\ )$ in S^4_i, where (i,j,k)=(1,2,3), (2,3,1), (3,1,2). We prove that, we have the equality $\beta(L_1)+\ \beta(L_2)+\ \beta(L_3) = 0$, where $\beta(L_i)$ is the Sato-Levine invariant of L_i, if all L_i are semi-boundary links.

1. Introduction and Main results

Take transverse immersions $f: S^4_1 \amalg S^4_2 \amalg S^4_3 \looparrowright S^6$ such that (1) $f|S^4_i$ is an embedding, (2) $f(S^4_i) \cap f(S^4_j) \neq \phi$ and $f(S^4_i) \cap f(S^4_j)$ is connected, and (3) $f(S^4_1) \cap f(S^4_2) \cap f(S^4_3) = \phi$. Then we obtain three surface-links L_i= $(f^{-1}(f(S^4_i) \cap f(S^4_j)),\ f^{-1}(f(S^4_i) \cap f(S^4_k))\ )$ in S^4_i, where (i,j,k)=(1,2,3), (2,3,1), (3,1,2). An orientation is given to each naturally. In this paper, we discuss which ones we obtain.

In order to state our theorems, we need some definitions.

We work in the smooth category. $S^4_i \cap S^4_j$ is a closed orientable connected surface and is oriented naturally. Hereafter, a *surface* will always mean a closed oriented connected surface unless otherwise stated.

A *surface-$(F_1, ..., F_\mu)$-link* is a submanifold $L = (K_1, ..., K_\mu)$ of S^4 such that K_i is diffeomorphic to the oriented surface F_i. If $\mu = 1$, L is called a *surface-F_1-knot.* A surface-(F_1, F_2)-link $L = (K_1, K_2)$ is called a *semi-boundary link* if $[K_i] = 0 \in H_2(S^4 - K_j; \mathbb{Z})$ $(i \neq j)$ ([18]). A surface-(F_1, F_2)-link $L = (K_1, K_2)$ is called a *boundary link* if there exist Seifert hypersurfaces V_i for K_i $(i = 1, 2)$ such that $V_1 \cap V_2 = \phi$. A surface-(F_1, F_2)-link (K_1, K_2) is called a *split link* if there exist 4-balls B^4_1 and B^4_2 in S^4 such that $B^4_1 \cap B^4_2 = \phi$ and $K_i \subset B^4_i$.

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Definition. (L_1, L_2, L_3) is called a *triple of surface-links* if L_1 is a (F_{12}, F_{13})-link, L_2 is a (F_{23}, F_{21})-link, L_3 is a (F_{31}, F_{32})-link, and F_{ij} is diffeomorphic to F_{ji} ((i,j)=(1,2),(2,3),(3,1)).

Definition. A triple of surface-links (L_1, L_2, L_3) is said to be *realizable* if there exists a transverese immersion $f : S^4_1 \amalg S^4_2 \amalg S^4_3 \looparrowright S^6$ such that (1) $f|S^4_i$ is an embedding (i=1,2,3), (2) $(f^{-1}(f(S^4_i) \cap f(S^4_j)), f^{-1}(f(S^4_i) \cap f(S^4_k))$) in S^4_i is L_i=(K_{ij}, K_{ik}) ((i, j, k)=(1,2,3), (2,3,1), (3,1,2)), and (3)$f(S^4_1) \cap f(S^4_2) \cap f(S^4_3) = \phi$.

We state the main theorem.

Theorem 1.1. *Let L_1, L_2 and L_3 be semi-boundary surface-links. Let (L_1,L_2,L_3) be a triple of surface-links. Suppose the triple of surface-links (L_1, L_2, L_3) is realizable. Then we have the equality*

$$\beta(L_1) + \beta(L_2) + \beta(L_3) = 0,$$

where $\beta(L_i)$ is the Sato-Levine invariant of L_i.

We review the Sato-Levine invariants in §2. Since there exists a triple of surface-links (L_1, L_2, L_3) such that $\beta(L_1)$=$\beta(L_2)$=0 and $\beta(L_3)$=1 (See §2.), we have:

Corollary 1.2. *Not all triple of surface-links are realizable.*

We prove:

Theorem 1.3. *There exists a realizable triple of surface-links (L_1, L_2, L_3) such that $\beta(L_1)$=1, $\beta(L_2)$=1, and $\beta(L_3)$=0.*

We prove the following sufficient conditions for the realization.

Theorem 1.4. *Let L_1, L_2 and L_3 be split surface-links. Let (L_1, L_2, L_3) be a triple of surface-links. Then the triple of surface-links (L_1, L_2, L_3) is realizable.*

Theorem 1.5. *Suppose L_i are (S^2, S^2)-links and L_i are slice links($i = 1, 2, 3$). Then the triple of surface-links (L_1, L_2, L_3) is realizable.*

We give problems.

Problem 1.6. *(1) Determine the realizable triple of surface-links.*

(2) Is the inverse of Theorem 1.1 valid?

(3) Let L_1, L_2 and L_3 be (S^2, S^2)-links. Then is the triple of surface-links (L_1, L_2, L_3) realizable?

Note. (i) Using a result of [15] (See §2.), one can show Problem 1.6.(3) follows from Problem 1.6.(2).

(ii) By Theorem 1.5, if the answer to Problem 1.6.(3) is negative, then the answer to an outstanding problem: "Is every (S^2, S^2)-link slice?" is negative. (Refer to [5], [6], and [12] for the slice problem.)

This paper is organized as follows. In §2 we review the Sato-Levine invariant. In §3 we prove Theorem 1.1. In §4 we prove Theorem 1.3. In §5 we prove Theorem 1.4. In §6 we prove Theorem 1.5.

2. The Sato-Levine invariant and spin cobordism

The Sato-Levine invariant is defined by Sato (in [18]) and Levine (unpublished) independently. It is easy to prove that the following definition is equivalent to theirs.

Definition. Let $L = (K_1, K_2)$ be a semi-boundary surface-(F_1, F_2)-link. Then there exist Seifert hypersurfaces V_i for K_i $(i = 1, 2)$ such that $V_i \cap K_j = \phi (i \neq j)$. Let v_i be the oriented normal bundle of V_i in S^4. Let F be the oriented closed surface $V_1 \cap V_2$. F need not be connected. Then the congruence $TS^4|_F \cong TF\oplus\ v_1|_F\oplus\ v_2|_F$ induces a spin structure σ on F. We define the *Sato-Levine invariant* $\beta(L)$ of L so that $\beta(L)$=$[(F, \sigma)]$ $\in \Omega_2^{\mathrm{spin}} \cong \mathbb{Z}_2$ for L. We call (F, σ) a *special surface for L*.

By [17] and [18] the following holds.

Theorem. *([17] and [18]) Let F_1 be an oriented closed connected surface not diffeomorphic to the 2-sphere. Let F_2 be an arbitrary oriented closed connected surface. Then there exists a semi-boundary (F_1, F_2)-link whose Sato-Levine invariant is one.*

In [15] Orr proved the following.

Theorem. *([15]) The Sato-Levine invariant of an arbitrary (S^2, S^2)-link is zero.*

The Sato-Levine invariant and its generalization are studied in [1], [2], [3], [4], [7], [8], [10], [11], [16], [19], [20], P.103 of [21], etc. [2] says that the Sato-Levine invariant is connected with [9].

3 The proof of Theorem 1.1.

Let $L_1 = (K_{12}, K_{13})$, $L_2 = (K_{23}, K_{21})$, and $L_3 = (K_{31}, K_{32})$. Let $f : S_1^4 \coprod S_2^4 \coprod S_3^4 \looparrowright S^6$ be an immersion to realize (L_1, L_2, L_3). We abbreviate $f(S_i^4)$ to S_i^4. We first prove:

Claim. *There exist Seifert hypersurfaces A_i for S_i^4 $(i = 1, 2, 3)$ such that $A_1\cap\ S_2^4\cap\ S_3^4$ $=\phi$, $A_2\cap\ S_3^4\cap\ S_1^4$ $=\phi$, and $A_3\cap\ S_1^4\cap\ S_2^4$ $=\phi$.*

Proof. Let $S_2^4\times\ D^2$ be a tubular neighborhood of S_2^4 in S^6. Put D^2=$\{(x, y)|\ x^2 + y^2 \leqq 0\}$. Then S_2^4=$S_2^4 \times \{(0, 0)\}$. Put I=$\{(x, y)|\ 0 \leqq x \leqq 1,\ y = 0\}$. We can regard $S_2^4\times\ D^2$ as the result of rotating $S_2^4 \times I$ around the axis S_2^4.

Put $M = (S_2^4 \times I)\ \cap S_1^4$. As we rotate $S_2^4 \times I$ as above, we rotate M as well. The result is $(S_2^4 \times D^2)\cap S_1^4$.

Take a Seifert hypersurface A_1' for S_1^4. Then $A_1' \cap S_2^4$ in S_2^4 is a Seifert hypersurface V_{21}' for K_{21}. We can suppose that $A_1' \cap (S_2^4 \times p)$ in $S_2^4 \times p$ is the submanifold V_{21}' for each $p \in D^2$.

Since $L_2 = (K_{23}, K_{21})$ is a semi-boundary link, there is a Seifert hypersurface V_{21} for K_{21} such that $V_{21} \cap K_{23}$=ϕ. Then there exists a compact oriented 4-manifold W in $S_2^4 \times I$ with the following properties.

(1) $W \cap S_2^4$ in S_2^4 is the submanifold V_{21}.
(2) $W\cap\ (S_2^4 \times \{(1, 0)\})$ in $(S_2^4 \times \{(1, 0)\})$ is the submanifold V_{21}'.
(3) $\overline{(\partial W) - V_{21}^2 - V_{21}'^2}$ is M.

When we rotate $S^4_2 \times I$ as above, we rotate W together. Let P denote what is made from W. Note that $\partial P = \partial(\overline{A'_1 \cap (S^4_2 \times D^2)}) = \partial(\overline{A'_1 - (A'_1 \cap (S^4_2 \times D^2))}) - S^4_1$. Let $A_1 = \overline{A'_1 - A'_1 \cap (S^4_2 \times D^2)} \cup P$.

Then $A_1 \cap S^4_2 \cap S^4_3 = V_{21} \cap K_{23} = \phi$. Note that, when we modify A'_1 to obtain A_1, we don't change f.

Replace (1,2,3) with (2,3,1) (resp. (3,1,2)) in the above proof. Then we obtain A_2 (resp. A_3). We now obtain A_1, A_2 and A_3 so that we keep the immersion f. This completes the proof. $\square$

Put $X = A_1 \cap A_2 \cap A_2$. Put $F_i = (\partial X) \cap S^4_i$. Then $\partial X = F_1 \amalg F_2 \amalg F_3$. By using A_1, A_2, A_3 and S^6, we give F_i (resp. W) a spin structure σ_i (resp. τ). Of course $\partial(X, \tau) = \amalg^3_{i=1}(F_i, \sigma_i)$. Then (F_i, σ_i) is a special surface for L_i. Therefore, $\Sigma_i \beta(L_i) = \Sigma_i[(F_i, \sigma_i)] = [\partial(X, \tau)] = 0 \in \Omega^{\mathrm{Spin}}_2$.

4 The proof of Theorem 1.3

Let $L_1 = (K_{12}, K_{13})$ be the (T^2, S^2)-link in [17]. Let $L_2 = (K_{23}, K_{21})$ be the (S^2, T^2)-link obtained by changing the order of L_1. Let $L_3 = (K_{31}, K_{32})$ be the trivial (S^2, S^2)-link. Note $\beta(L_1) = \beta(L_2) = 1$ and $\beta(L_3) = 0$. It suffices to prove that the triple of surface-links (L_1, L_2, L_3) is realizable.

(K_1, K_2) is called a *pair of surface-F-knots* if both K_1 and K_2 are F-knots. A pair of F-knots (K_1, K_2) is said to be *realizable* if there exists a transverse immersion $f : S^4_1 \amalg S^4_2 \looparrowright S^6$ such that (1)$f|S^4_i$ is an embedding $(i = 1, 2)$, and (2) $f^{-1}(f(S^4_1) \cap f(S^4_2))$ in S^4_i is $K_i (i = 1, 2)$.

We prove:

Proposition 4.1. *Let K be a surface-knot. Then the pair of surface-knots (K, K) is realizable.*

Take an embedding $f : S^4_1 \coprod S^4_2 \hookrightarrow S^6$. There exists a chart U of S^6 such that

(1) $\phi : U \cong \mathbb{R}^4 \times \{(u, v) | u, v \in \mathbb{R}\} \cong \mathbb{R}^4 \times \mathbb{R}_u \times \mathbb{R}_v$, and

(2)$U \cap f(S^4_1) = \mathbb{R}^4 \times \{(u, v) | u = 0, v = 0\}$. Call it $\mathbb{R}^4_1$.

$U \cap f(S^4_2) = \mathbb{R}^4 \times \{(u, v) | u = 1, v = 0\}$. Call it $\mathbb{R}^4_2$.

We prove Lemma 4.2. Obviously it induces Proposition 4.1.

Lemma 4.2. *There exists a transverse immersion $g : S^4_1 \coprod S^4_2 \looparrowright S^6$ to realize the pair of surface-knots (K, K) with the following properties.*

(1) $g|S^4_2 = f|S^4_2$.
(2) $g(S^4_2) \cap \mathbb{R}^4 \times \{u = 0\} \times \mathbb{R}_v = g(S^4_2) \cap \mathbb{R}^4 \times \{u = 0\} \times \{v = 0\}$.
(3) $g|S^4_1$ *is isotopic to* $f|S^4_1$.

We modify the embedding f to obtain an immersion g.

Take any Seifert hypersurface V for K in $\mathbb{R}^4_1$. Let $N(V) = V \times \{t | -1 \leqq t \leqq 1\}$ be a tubular neighborhood of V in $\mathbb{R}^4_1$. We define the subset E of $N(V) \times \mathbb{R}_u \times \mathbb{R}_v$
$= \{(p, t, u, v) | p \in V, -1 \leqq t \leqq 1, u \in \mathbb{R}, v \in \mathbb{R}\}$ so that

$$E = \{(p, t, u, v) |\ p \in V, \quad 0 \leqq u \leqq \tfrac{\pi}{2}, \quad t = k \cdot \cos u, \quad v = k \cdot \sin v, \quad -1 \leqq k \leqq 1\}.$$

Put $X = \overline{(\partial E) - N(V)}$ and $Y = \overline{f(S_1^4) - N(V)}$. Then $\partial X = \partial Y = \partial N(V)$. Put $\Sigma = X \cup Y$. Then Σ is an embedded 4-sphere. We define $g|S_1^4$ so that $g(S_1^4) = \Sigma$. This completes the proof of Lemma 4.2 and therefore Proposition 4.1.

Note. See Figure 4.1 We draw a lower dimensional analogue. There, we replace $\mathbb{R}^4 \times \mathbb{R}_u \times \mathbb{R}_v$ with $\mathbb{R}^2 \times \mathbb{R}_u \times \mathbb{R}_v$.

Figure 4.1 is divided into the three pieces.

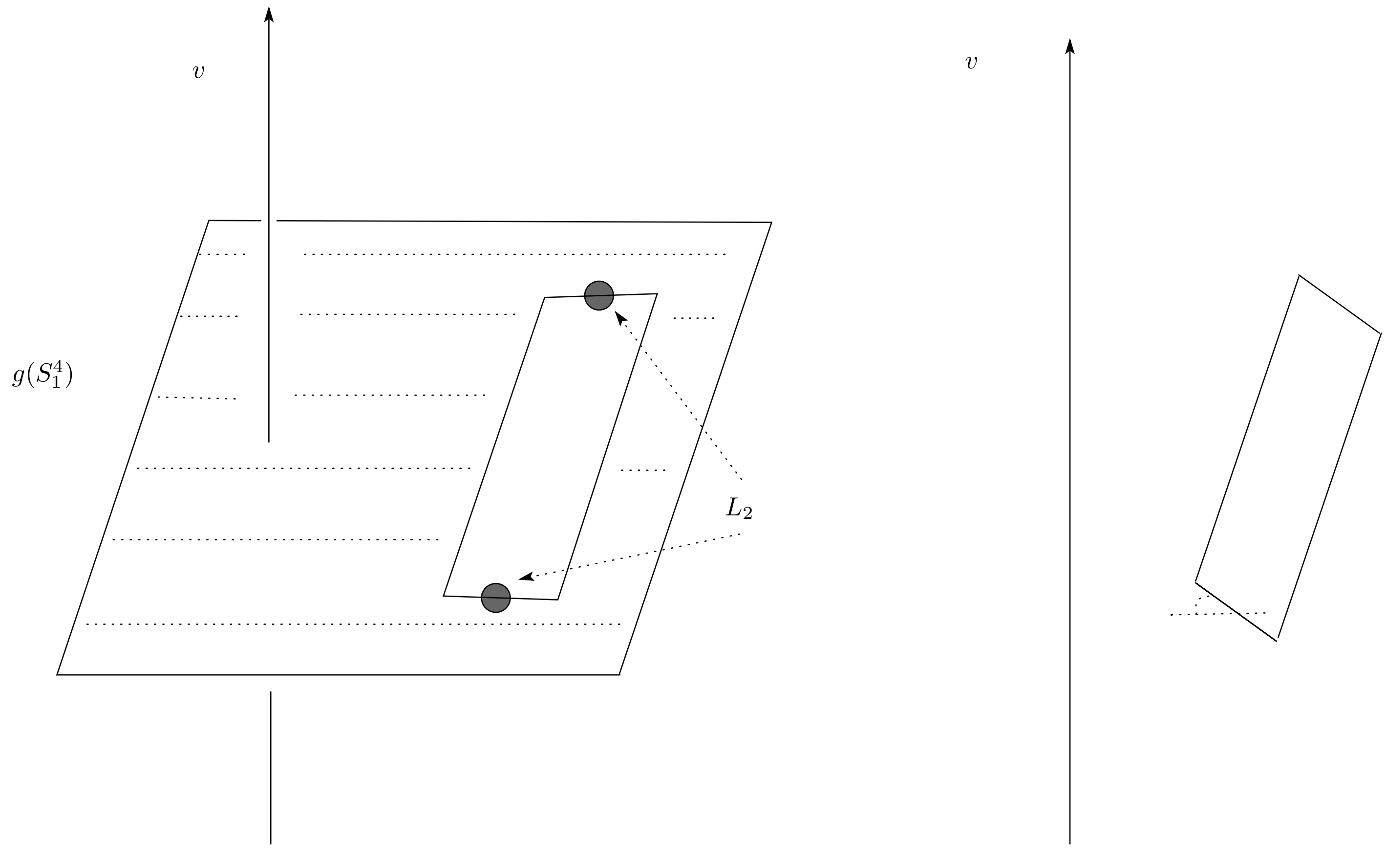
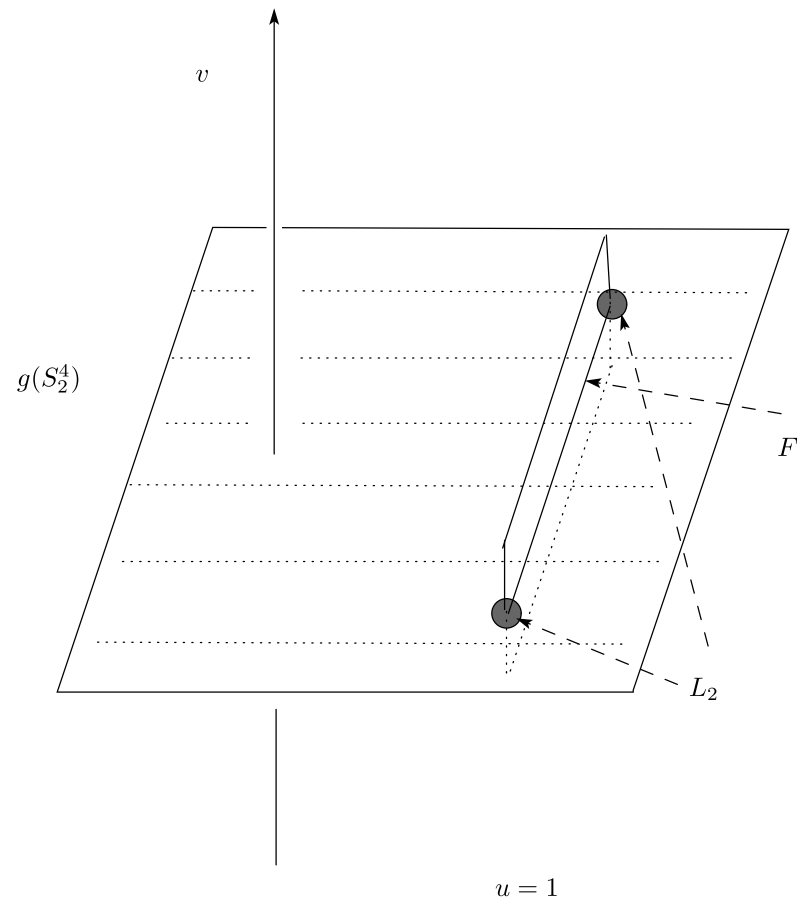


Figure 4.1(the left of the three)



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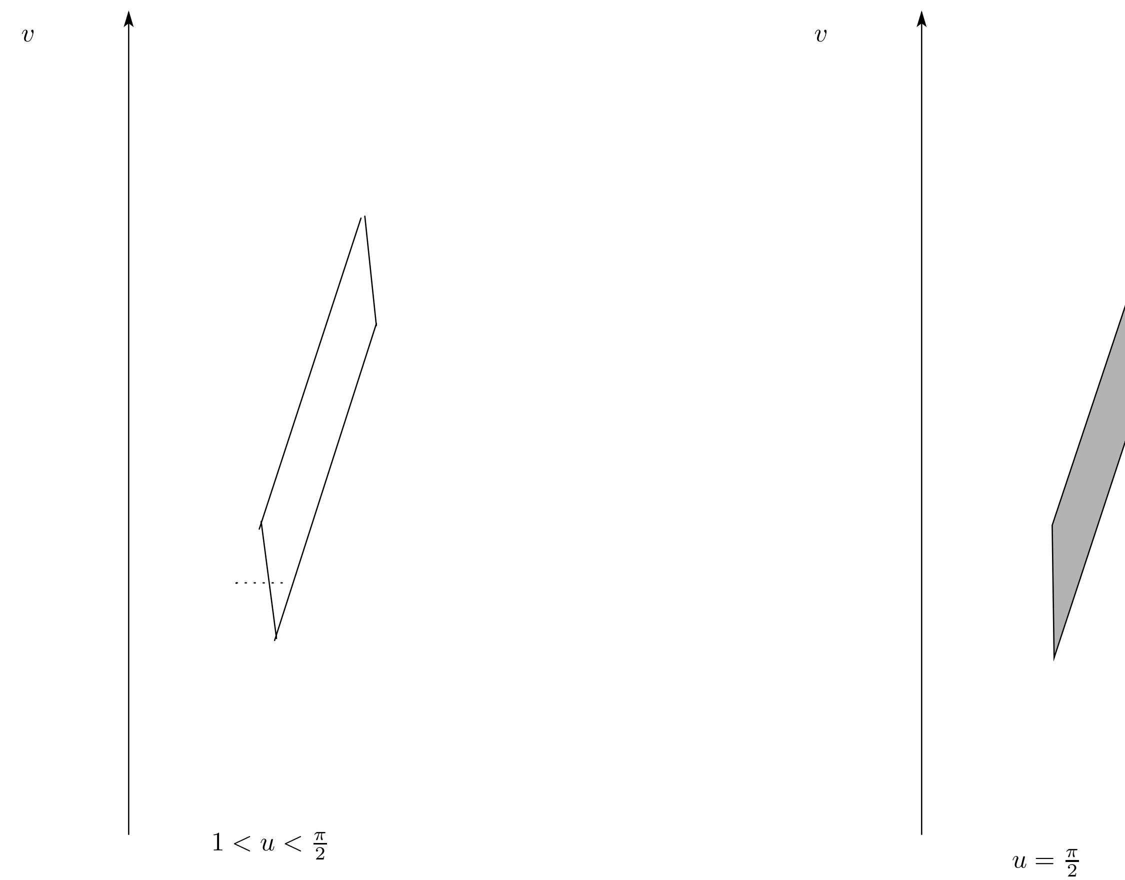

Figure 4.1(the right of the three)

Figure 4.1 is divided into the three pieces.

By the definition of L_i, the T^2-knots K_{12} and K_{21} are equivalent. Therefore there is an immersion $g : S^4_1 \coprod S^4_2 \looparrowright S^6$ to realize the pair of T^2-knots (K_{12}, K_{21}).

We prove the following Lemma 4.3. Obviously Lemma 4.3 induce Theorem 1.3.

Lemma 4.3. *There exists a transverse immersion $h : S^4_1 \coprod S^4_2 \coprod S^4_3 \looparrowright S^6$ to realize (L_1, L_2, L_3) with the following properties.*

(1) $h|_{S^4_1 \quad S^4_2} = g$

(2) $h(S^4_3) \subset U$. *$h(S^4_3)$ is the trivial 3-knot.*

Proof. We modify the immersion g to obtain an immersion g.

Take K_{13} (resp. K_{23}) in $\mathbb{R}^4_1$ (resp. $\mathbb{R}^4_2$). There is a Seifert hypersurface V_{12} for K_{12} so that $V_{12} \cap K_{13} = \phi$. Take V_{12} as a Seifert hypersurface used in the proof of Lemma 4.2. Recall V_{12} and K_{13} are in $\mathbb{R}^4_1$.

Recall K_{13} and K_{23} are the trivial S^2-knots. Take a 3-ball B^3_{13} (resp. B^3_{23}) which bounds K_{13} (resp. K_{23}) in $\mathbb{R}^4_1$ (resp. $\mathbb{R}^4_2$). Note that B^3_{13} does not include in $g(S^4_1)$.

Take the 5-ball B^5= $\{(q, u, v)|q \in B^3,\ -1 \leqq u \leqq 2, -2 \leqq v \leqq 2\}$ in U. Suppose $B^5 \cap \mathbb{R}^4_1$=B^3_{13} and $B^5 \cap \mathbb{R}^4_2$=B^3_{23}. Then $(\partial B^5)$$\cap$ $S^4_1 \cap S^4_2$=ϕ.

Define $h|S^4_3$ so that $h(S^4_3)$=∂B^5.

This completes the proof of Lemma 4.3 and hence Theorem 1.3.

5 The proof of Theorem 1.4 and a relation between knot cobordism and the realization of pair of knots

Surface-F-knots K_0 and K_1 are said to be *cobordant* or *concordant* if there is a smooth submanifold W of $S^4 \times [0, 1]$, which meets the boundary transversely in ∂W, is diffeomorphic to $F \times [0, 1]$ and meets $S^4 \times \{i\}$ in K_i $(i = 0, 1)$.

We prove the following although it may be folklore.

Theorem 5.1. *Let F be a closed connected oriented surface. Then arbitrary F-knots K_0 and K_1 are cobordant.*

Proof. Let L be a split surface-link with components K_0 and $-K_1$. It suffices to prove:

Claim. *There exists a submanifold of S^4 which is diffeomorphic to F×[0,1] such that F×[0,1] intersects with ∂B^5 transversely, F×[0,1] $\cap$ ∂B^5 $= F \times \{0\} \amalg F \times \{1\}$, and ($F \times \{0\}$, $F \times \{1\}$) in S^4=∂B^5 is L.*

Let V be a connected Seifert hypersurface for L. A spin structure on V is induced from the unique one on S^4. A spin structure on ∂V is induced from the one on V. Make a closed spin 3-manifold $W = V \cup (F \times [0, 1])$ so that the spin structure on V extend to the one on W. Note W is not a submanifold of S^4. Since $\Omega^{\mathrm{spin}}_3 = 0$, there exists a spin 4-manifold X which W spin-bounds. Since V and $F \times [0, 1]$ are connected, we can take a handle decomposition $X = (V \times [0, 1])$$\cup$(4-dimensional 2-handles h^2) $\cup\{(F \times [0, 1]) \times [0, 1]\}$. Take V× [0,1] in S^4 × [0,1] so that V × $\{t\}$ is in S^4 × $\{t\}$.

Attach the handles h^2 to $V \times \{1\} \subset S^4 \times \{1\}$. Then we can attach the 5-dimensional 2-handles $\bar{h}^2$=$h^2 \times [-1,1]$ to S^4 $\times\{1\}$ naturally. Let Y= S^4 $\times$[0,1] $\cup$(the 5-dimensional 2-handles $\bar{h}^2$). Since the attaching maps of $\bar{h}^2$ are spin preserving diffeomorphisms, Y is diffeomorphic to ($\natural^* S^2 \times B^3$)$-$(the 5-ball). ∂Y is a disjoint union of the standard 4-sphere S^4_0 and ($\sharp^* S^2 \times S^2$). Hence Y is embedded in B^5 so that S^4_0 coincides with ∂B^5.

Therefore $F \times [0,1] \subset W \subset B^5$ and the submanifold $F \times [0,1]$ satisfies the condition in the Claim. This completes the proof. $\square$

It is easy to prove that Theorem 1.4 is equivalent to the following Theorem 5.2. We prove:

Theorem 5.2. *Let F be a closed connected oriented surface. If F-knots K and K' are cobordant, the pair of F-knots (K, K') is realizable.*

Proof. By Proposition 4.1, the pair of F-knots (K, K') is realizable. Hence it suffices to prove:

Claim. *Suppose that a pair of F-knots (K_1, K_2) is realizable. Suppose that K_2 is cobordant to K_3. Then (K_1, K_3) is realizable*

Proof. Let $f : S^4_1 \coprod S^4_2 \looparrowright S^6$ be an immersion to realize (K_1, K_2). We construct an immersion $\widetilde{f} : S^4_1 \coprod S^4_2 \looparrowright S^6$ to realize $(K_1,\, K_3)$ as follows. Put $\widetilde{f}|S^4_2 = f|S^4_2$.

Let $f(S^4_2) \times D^2$ be a tubular neighborhood of S^4_2 in S^6. Put D^2=$\{(x,y)|\, x^2$+$y^2 \leqq 0\}$. Then $f(S^4_2)$=$S^4_2 \times \{(0,0)\}$. Put I=$\{(x,y)|\ 0 \leqq x \leqq 1,\ y = 0\}$. We can regard $f(S^4_2)\times$ D^2 as what is obtained by rotating $f(S^4_2) \times I$ around $f(S^4_2)$ as the axis.

Put $M = (f(S^4_2) \times I)\ \cap f(S^4_1)$. We can regard $(f(S^4_2) \times D^2)\ \cap f(S^4_1)$ as what is made from M as follows: When we rotate $(f(S^4_2) \times I)$ as above, we rotate M together. What is made from M is $(f(S^4_2) \times D^2) \cap f(S^4_1)$.

We can suppose that $\{f(S^4_2 \times p)\} \cap f(S^4_1)$ in $f(S^4_2) \times p$ is K_2 for each $p \in D^2$.

Since K_2 and K_3 are cobordant, there is a compact oriented 3-manifold P in $f(S^4_2)\times I$ with the following properties. (1) $P \cong F\times[0,1]$. (2) P intersects $f(S^4_2)\times\partial I$ transversely. $P\ \cap f(S^4_2)$ in $f(S^4_2)$ is K_3. $P\cap[f(S^4_2) \times \{(1,0)\}]$ in $f(S^4_2) \times \{(1,0)\}$ is K_2.

When we rotate $f(S^4_2) \times I$ as above, rotate P together. Let Q denote what is made from P.

Note that ∂Q =∂ $\overline{f(S^4_1) \cap (f(S^4_2) \times D^2)}$) =$\partial$ $\overline{f(S^4_1) - (f(S^4_1) \cap (f(S^4_2) \times D^2)}$). Then R=$\overline{f(S^4_1) - f(S^4_1) \cap (f(S^4_2) \times D^2)}$ $\cup Q$ is a 4-sphere embedded in S^6. Put $\widetilde{f}(S^4_2) = R$. This completes the proof.

6 The proof of Theorem 1.5.

It is easy to prove that it suffices to prove:

Proposition. *Let $L = (K_1, K_2)$ be a (S^2, S^2)-link and a slice link. Then there exists three 4-spheres S^4_1, S^4_2, and S^4 embedded in S^6 with the following properties.*

(1)$S_1^4 \cap S_2^4 = \phi$ *(2)*$(S_1^4 \cap S^4, S_2^4 \cap S^4)$ *in* S^4 *is* L*.*

Proof. Let $S^4\times\, D^2$ denote a tubular neighborhood of S^4 in S^6. Put D^2=$\{(x,y)|$ $x^2+y^2 \leqq 0\}$. Then S^4=$S^4 \times \{(0,0)\}$. Put I=$\{(x,y)|\ 0 \leqq x \leqq 1,\, y=0\}$. We can regard $S^4\times\, D^2$ as the result of rotating $S^4 \times I$ around the axis S^4.

Since the 2-link L is slice, there exists two 3-discs D_1^3 and D_2^3 in $S^4 \times I$ with the following properties. (1) $D_1^3 \cap D_2^3$=ϕ. (2) D_i^3 intersects S^4 transversely. D_i^3 $\cap S^4$=∂D_i^3. (3) (∂D_1^3, ∂D_2^3) in S^4 is the 2-link L.

When we rotate $S^4 \times I$ as above, we rotate D_1^3 $\amalg D_2^3$ together. This gives 4-spheres S_1^4 and S_2^4 embedded in S^6. This completes the proof.

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